THE BASIC ARITHMETIC OF LEGISLATIVE DECISIONS*

Michael Laver
New York University
michael.laver@nyu.edu

Kenneth Benoit
London School of Economics
and Trinity College Dublin
kbenoit@lse.ac.uk

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ABSTRACT

Despite the huge number of possible seat distributions following a general election in a multi-party parliamentary democracy, there are far fewer equivalence classes of seat distribution sharing important strategic features. We define an exclusive and exhaustive partition of the universe of theoretically-possible *n*-party systems into five basic types, the understanding of which facilitates more fruitful modeling of legislative politics, including government formation. A common type of legislative party system has a "strongly-dominant" party in the privileged position of being able to play off the other parties against each other. Another is a "top-three" party system in which the three largest parties are perfect substitutes for each other in the set of winning coalitions, but no other party is ever pivotal. Having defined a partition of legislative party systems and elaborated logical implications of this partition, we classify a large set of postwar European legislatures. We show empirically that many of these are close to critical boundary conditions, so that the stochastic processes involved in any legislative election could easily flip the resulting legislature from one type to another. This is of more than hypothetical interest, since we also show that important political outcomes differ systematically between the basic party system types – outcomes that include the duration of government formation negotiations, the type of coalition cabinet that forms, and the stability of the resulting government.

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INTRODUCTION

In multi-party legislatures where no single party controls a winning seat share, making and breaking governments, as well as passing bills and resolutions, requires finding winning voting coalitions. A vast number of possible distributions of seats between parties follow any legislative election in a multiparty system. Even ignoring party names, for example, there are 38,225 different distributions of 100 seats between five parties, and 2,977,866 such distributions between ten parties (Laver and Benoit 2003). The number of possible coalitions increases at an exponential rate as the number of parties increases. Notwithstanding the profusion of superficially different possibilities that politicians must evaluate, many of these are functionally equivalent. This allows us to define a set of equivalence classes that highlights basic similarities within classes of legislature, bearing on the building of winning coalitions. This highlights a plain fact of *realpolitik* that, when a legislature is close to a boundary between classes, small shocks to the seat distribution may have big effects on legislative politics. Conversely, when a legislature is far from such a boundary, big shocks may have little effect. For any actual legislature, therefore, it is important to know which equivalence class it is in, and how close to a boundary condition it is located.

More generally, since any observed legislative election result is the realization of a random draw from a *distribution* of expected election results, different draws from the same distribution may result in legislatures that fall into different equivalence classes – making a big difference to legislative politics. When the distribution of expected election results straddles a boundary between classes of legislature, small changes in the number of seats held between parties can flip the realized legislature from one class to another, making the effective election result, in terms of downstream politics, something of a dice roll. Following the realization of an actual election result, furthermore, *non-random* strategic defection of legislators from one party to another may flip the legislature from one class to another, any may therefore offer potentially greater payoffs if the legislature is close to a boundary condition.

Motivating this discussion with a simple example, consider a three-party 100-seat legislature with a majority winning threshold and a seat distribution of (49, 49, 2). A tiny shock to seat shares may transform legislative politics; (50, 49, 1) and (51, 49, 0) are both very different legislatures. At the same time, either or both of the legislators in the smallest party may have a huge effect, from which they may extract considerable rents, by defecting to one of the other parties. The situation is completely different if the seat distribution is (34, 33, 33). This is despite the fact that both legislatures fall into the same equivalence class in one important sense:

any two parties, but no single party, can form a winning coalition, so that all parties have the same theoretical voting weight.[1] Given a majority winning threshold, the (49, 49, 2) legislature is much closer to a critical boundary condition – something that is ignored if we focus only on theoretical voting weights.[2]

It has long been known that big discontinuities in legislative politics may arise from small changes in the legislative arithmetic. If a proposal is supported by a legislative coalition one vote short of the winning threshold then the outcome, defeat, is in many respects the same as if the support coalition had been 100 votes short. A quite different outcome, victory, arises if the support coalition has a single extra vote and reaches the winning threshold. The strategic implications of such thresholds have not passed unnoticed. Within the traditions of cooperative game theory, they give rise to notions such as the Shapley value and power indices such as the Shapley-Shubik and Banzhaf indices (Banzhaf 1965; Shapley and Shubik 1954; Shapley 1952; Felsenthal and Machover 1998; Stole and Zwiebel 1996).[3] Many different distributions of seats between parties generate the same vector of Shapley or Banzhaf values. For example, the set of theoretically possible five-party 100-seat legislatures referred to above has 38,225 different distributions of seats between parties, but only 20 different Shapley vectors (Laver and Benoit, 2003). Shifting a single seat from one party to another can change the Shapley values dramatically, or not change them at all. Within the traditions of non-cooperative game theory, these thresholds inform a literature on "minimal integer representations" (MIRs) of weighted voting games, which depend on the fact that many possible distributions of legislative seats between parties generate the same set of winning coalitions (Ansolabehere et al. 2005; Laver et al. 2011; Montero 2006; Snyder et al. 2005; Freixas and Molinero 2009).[4]

We have three core objectives in this paper. First, we specify a partition of legislative party systems into far fewer equivalence classes than Shapley vectors or MIRs and derive theoretically relevant implications of this classification. Second, we show empirically that many legislative party systems in postwar Europe are close to critical boundary conditions. This means

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[1] We say what we mean by "theoretical voting weight" below. The two legislatures in the example have the same minimal integer representation and Shapley vector.
[2] The minimal integer representation and Shapley vectors for both legislatures are identical.
[3] Stole and Zwiebel (1996), among others, derive the Shapley value as a prediction from a non-cooperative alternating offers bargaining game.
[4] A minimal integer representation is the vector of smallest integers that generates, for a given winning quota, the same set of winning coalitions as the vector of raw seat totals. Consider three very "different" legislative party systems in a setting with a majority decision rule: (49, 17, 17, 17); (27, 25, 24, 24); and (2, 1, 1, 1). All generate the same set of winning coalitions. The largest party can form a winning coalition with any other; all others must combine to exclude the largest party. These legislative party systems share the same vector of Shapley or Banzhaf values (1/2, 1/6, 1/6, 1/6), and the same MIR (2, 1, 1, 1). Despite large superficial differences, in this precise sense these party systems are in an equivalence class.

that the random shocks arising from stochastic processes associated with any real election could easily flip the resulting legislature from one equivalence class to another. Third we show that this is substantively important, since different political outcomes are associated in real parliamentary democracies with different types of legislature. First, however, we motivate our argument with a recent example of government formation where our boundary conditions made a big difference.

GREECE 2012

Greek voters went to the polls in May 2012 facing the specter of default on their country's sovereign debt. With markets plunging in anticipation of a possible unraveling of the Eurozone should the resulting Greek government not accept terms of an EU-led bailout package, the election hinged on whether its result would enable the formation of a pro-bailout government. In the event the largest party, New Democracy (ND), won just 108 of the 300 legislative seats, 43 short of the majority needed to form a government (see Table 1). The only two-party winning coalition was between the ND and the second largest party, Syriza. This generated what we below call a "top-two" party system, complicated by the fact that the top two parties fundamentally disagreed on the key issue of the election, the EU bailout. ND approached every other party except the extreme anti-European Golden Dawn (XA). Each refused to go into government.

May		June	
Name	Seats	Name	Seats
ND	108	ND	129
Syriza	52	Syriza	71
PASOK	41	PASOK	33
ANEL	33	ANEL	20
KKE	26	XA	18
XA	21	DIMAR	17
DIMAR	19	KKE	12
Total	300		300
Threshold	151		151
Legislative type	D		B

Table 1. *Legislative arithmetic in the Greek elections of May and June 2012 and 2010.* "Legislative Type" is explained below.

As mandated by the Greek constitution, the second largest party (Syriza) and third largest (PASOK) each in turn attempted to form governments. These attempts also failed. As a last resort, the President himself proposed a government comprising ND, PASOK and a small left wing party, Democratic Left (DIMAR). However DIMAR, from the beginning reluctant to accept conditions of the EU-IMF package, blocked this, knowing ND and PASOK lacked the 151 seats needed to form a government, even though they were only two seats short of this.

New elections were called for June. A new roll of the dice produced a nearly identical ordering of parties, but one crucial difference in the basic legislative arithmetic. The first and third largest parties, two seats short after the previous election, now controlled a majority of seats between them. The Greek legislative party system had therefore flipped out of a "top two" state and had made ND a "strongly dominant" party. This substantially weakened the second largest party, Syrizia, *despite the fact that Syrizia had increased its seat total from 52 to 71*. The key fact arising from the new legislative arithmetic in Greece was that that ND and PASOK could now form a government alone – *despite the fact that the ND seat total had declined from 41 to 33*. Given the new reality that the anti-bailout Syriza could not form a government even with all of the other parties, DIMAR accepted the deal they had blocked one month before, joining the government with "conditional support".[5] Had the May outcome resulted in just two more seats for the ND-PASOK coalition, making ND a dominant party and flipping the Greek party system into Type B, in the sense we define below, the pro-bailout Greek government might well have been formed a month before, sparing weeks of financial turmoil, market losses, and political crises. Given the Greek electoral system, furthermore, a 0.5% perturbation in the vote shares could easily have resulted in this two-seat shift. A fundamental constraint on government formation following the May 2012 Greek election was in effect determined by a dice roll which would flip the Greek party system from one state to another on the basis of what were essentially small random perturbations consistent with the same underlying conditions.

BASIC TYPES OF LEGISLATIVE PARTY SYSTEM

An exclusive and exhaustive partition of the universe of legislative party systems

Consider a legislative party system consisting of N perfectly disciplined and exogenously selected[6] parties holding seats in a legislature of M total seats. The set of parties, in descending order of seat share, is $P_1, P_2, \ldots P_n$. The number of seats controlled by P_i is S_i. Any legislative party system can be written as (W: S_1, S_2, ... S_n). According to binding constitutional rules, a successful proposal must be supported by a coalition of legislators whose number equal or

[5] Reuters Jun 19, 2012, 01.05PM IST (2012-06-19). "Greece elections: Conservative New Democracy poised to clinch coalition deal with PASOK - Economic Times". Economictimes.indiatimes.com.

[6] A perfectly-disciplined legislative party is a set of legislators who, for unmodeled reasons to do with intra-party politics, always vote the same way on any matter.

exceeds W.[7] The winning quota is decisive: if a coalition, C, of legislators is winning then its complement, C', is losing. This means that W must be *at least* a simple majority of legislators, though it is important to note that in most of what follows W could also be a supermajority.[8] We denote a coalition between P_x and P_y as P_xP_y. A "pivotal" party is one that can turn a losing coalition into a winning one by joining it.

We define a mutually exclusive and exhaustive partition of the universe of possible legislative party systems into five basic equivalence classes, which we will refer to as "types". We do this using the sizes of the three largest parties, relative to each other and to W. This partition is set out in Figure 1, although we note below an additional variant of Type B that distinguishes N-party systems with what we call " system-dominant" parties. We show below that, moving from Type A to Type E, we find a progressively weaker role for the largest party. In the Greek examples referred to in Table 1, for instance, the May elections produced a Type D ("top-two") system in which P_1P_2 is the *only* two-party winning coalition. The June elections, in contrast, produced a Type B system that removed the constraint blocking the winning P_1P_3 coalition that in the event formed.

<table>
<tr><th colspan="5">Universe of possible legislative party systems</th></tr>
<tr><td>Single winning party</td><td colspan="4">No single winning party</td></tr>
<tr><td rowspan="4">$S_1 \geq W$</td><td colspan="4">$S_1 < W$</td></tr>
<tr><td colspan="3">$S_1 + S_2 \geq W$</td><td rowspan="3">$S_1 + S_2 < W$</td></tr>
<tr><td colspan="2">$S_1 + S_3 \geq W$</td><td rowspan="2">$S_1 + S_3 < W$</td></tr>
<tr><td>$S_2 + S_3 < W$</td><td>$S_2 + S_3 \geq W$</td></tr>
<tr><td>A: Single winning party</td><td>B: Strongly dominant party</td><td>C: Top-three</td><td>D: Top-two</td><td>E: Open</td></tr>
</table>

Figure 1. *Partitioning the universe of legislative party systems.*

[7] In settings with a status quo, abstentions constitute *de facto* support for (opposition to) a proposal iff, had the abstainer voted nay (or yea), an otherwise winning (losing) proposal would have lost (won).

[8] Note immediately that if W is decisive, $S_1 + S_2 + S_3 < 2W$ and it must therefore be true that $S_2 + S_3 \leq 4W/3$. We will find this useful to know.

Definitions and properties of classes of legislative party system

Type A: Winning party

In Type A systems, a "majority" party $S_1 \geq W$, which must be the largest, controls all legislative decisions.[9] Trivially, if W is decisive and $S_1 \geq W$, then $S_2 < W$.

Type B: Strongly dominant party

Strongly dominant party systems are those in which P_1 has too few seats to control decisions ($S_1 < W$), but can form a winning majority with either of P_2 or P_3 ($S_1 + S_3 \geq W$), while P_2 and P_3 together cannot form a coalition that is winning ($S_2 + S_3 < W$). This makes P_1 "dominant" in the manner defined by previous authors (Peleg 1981; Einy 1985; van Deemen 1989): A party P is dominant if there is at least one pair of mutually exclusive losing coalitions excluding P, each of which is winning if P joins, but which cannot combine with each other to form a winning coalition. The key intuition derives from the arithmetical certainty that P, which must be the largest party, can win by joining with either losing coalition, while these cannot combine to exclude P. This allows P to play off each losing coalition against the other. Laver and Benoit (2003) show that dominant parties tend strongly to be pivotal members of more winning coalitions than are non-dominant parties *of the same size*. They therefore tend to have higher Shapley values than non-dominant parties of the same size and to have "super-proportional" expectations – their normalized Shapley values exceed their seat shares. Laver and Benoit also show that a dominant party's seat share must exceed half the winning threshold,[10] the first of a number of results that focus attention on the threshold $W/2$ in addition to W.

The definition of dominant party refers to mutually exclusive losing *coalitions* made winning by adding the largest party, but the intuition is more striking if we consider individual losing *parties*. We call party P^* "*strongly* dominant" if there are two other parties P_i and P_j such that $S_1^* + S_i \geq W$ and $S_1^* + S_j \geq W$ but $S_i + S_j < W$. The strongly dominant party is made dominant by joining losing *parties* to form winning coalitions, as opposed to joining losing *coalitions*. Since any party can be described as a singleton coalition, strongly dominant parties are special cases of dominant parties. Define Type B legislative party systems as those containing a strongly dominant party. There are several striking logical implications of having a

[9] If the party affiliation of legislators is endogenous, a legislature controlled by a single winning party may not be in steady state.

[10] Consider a pair of mutually exclusive losing coalitions, (C C^*), each of which excludes P_1 but can be made winning by adding $P_{1.}$ P_1 is dominant by definition *iff* $S_c + S_{c^*} <$ W and $S_{1\,+}\, S_c \geq W$ and $S_1 + S_{c^*} \geq W$. Imagine $S_1 < W/2$. This implies $S_c > W/2$ and $S_{c^*} > W/2$. This implies $S_c + S_{c'} > W$. Contradiction. It must be that $S_1 \geq W/2$ if P_1 is dominant.

strongly dominant party, which are "model free" in the sense they arise from binding arithmetic constraints and hold regardless of the utility functions of key agents or the local institutional structure.

> **Proposition B1: The sizes of the three largest parties determine whether P_1 is strongly dominant (the size of any other party has no bearing on this).** If two smaller parties, P_i and P_j, render P_1 strongly dominant, then P_2 and P_3 also render P_1 strongly dominant.[11] The inequalities $S_1 + S_3 \geq W$ and $S_2 + S_3 < W$ are necessary and sufficient conditions for P_1 to be strongly dominant.
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> **Proposition B2: If P_1 is strongly dominant, then both P_2 and P_3 must be members of any winning coalition excluding P_1.**[12] The special position of a strongly dominant P_1 imposes severe constraints on any coalition excluding it, which must include both second and third largest parties.
>
> **Proposition B3: P^* and only P^* is a member of every winning two-party coalition.**[13] This is another aspect of the privileged position of a strongly dominant party.
>
> **Proposition B4: If P_1 is strongly dominant, then $S_3 < W/2$.**[14] This is the second result focusing attention on $W/2$.

Simple arithmetical constraints on legislative decision-making give us powerful intuitions about the distinguished position of a strongly dominant party, should one exist. If P^* is excluded from any winning coalition, then both P_2 and P_3 must be members of this. But P^* can form a winning coalition with either P_2 or P_3 and indeed any two-party winning coalition must include P^*. As a consequence, P^* can make offers to both P_2 and P_3, to induce them to break any winning coalition excluding it, and these offers can be implemented by the winning coalitions P^*P_2 and P^*P_3 *without recourse to any other party*. Only a strongly dominant party can be in this privileged position. We show below that these results are empirically relevant because settings

[11] Since $S_2 \geq S_3 \geq S_i \geq S_j$, if the first two conditions strong dominance hold for S_i and S_j, they hold *a fortiori* for S_2 and S_3. To see that the third condition also holds, note that if P_1P_j is winning then its complement (P_1P_j)' is losing. For any $j > 3$, P_2P_3 is a subset of (P_1Pj)' and thus $S_2 + S_3 < W$. Thus, if the defining inequalities of strong dominance are fulfilled for any P_1, P_i and P_j, they are fulfilled for P_1, P_2 and P_3.

[12] Since the coalition P_1P_2 is winning by definition of strong dominance, its complement $(P_1P_2)'$ is losing. Thus $(P_1P_2)'$ must add either P_1 or P_2 to become winning. If it excludes P_1 it must add P_2. Thus if P_1 is strongly dominant, any winning coalition excluding P_1 must include P_2. An identical argument applies to P_3.

[13] Since the largest possible two-party coalition excluding P_1, which is P_2P_3, is losing, then every possible two-party coalition excluding P_1 is losing.

[14] If $S_2 + S_3 < W$ and $S_2 \geq S_3$, then $S_3 < W/2$.

with a strongly dominant party are not only common in postwar Europe, but also tend to be associated with minority governments.

Type B: System-dominant party*

A special case of a strongly dominant party occurs when the largest party P_1 is not winning on its own ($S_1 < W$) but can form a winning coalition with *any* other party ($S_1 + S_n \geq W$).[15] Call such a party, P^{**}, "system-dominant".

Proposition B2*: Any winning coalition excluding P^{**} must include all other parties. This is a necessary and sufficient condition for system dominance.[16]

This implies a strategic setting described by game theorists as an "apex game". Identifying Type B* party systems is useful theoretically because, moving beyond three parties, apex games have a structure that is more tractable analytically than many others (Fréchette et al. 2005a; Montero 2002). For example, it is easy to calculate both the Shapley vector and MIR for any *n*-party system with a system dominant party.[17] Identifying Type B* legislative party systems is important empirically because, as we show below, these tend to be associated with significantly shorter government formation negotiations, with single party minority cabinets, and with longer cabinet durations.

Type C: "Top-three" party system

While the sizes of the three largest parties determine whether there is a strongly dominant party, they also determine another important threshold. A "top-three" (Type C) legislative party system arises when $S_1 < W$, but *any* pair of the three largest parties can form a winning coalition. There is no dominant party in a top-three party system, for which $S_2 + S_3 \geq W$ is the single defining inequality.[18] Logically, this implies:

[15] This implies $S_2 + S_3 < W$ since P_2P_3 is in the (losing) complement of P_1P_n, for $n > 3$.

[16] For example, in a 100-seat legislature with a simple majority rule, this would arise if the partition of seats between 6 parties was (40, 12, 12, 12, 12, 12)

[17] Since a system-dominant party can only not be pivotal in the first and last position in any ordering of parties, and must therefore be pivotal in every ordering in which it is in one of the n-2 other positions, its normalized Shapley value must be (n-2)/n. The combined Shapley values of the other parties must be 2/n. Since the n-1 other parties are all in the identical position that any one of them can form a majority with the system dominant party but all must all combine to exclude it, by symmetry each non-dominant party must have a normalized Shapley value of 2/n·(n-1). The Shapley vector for any n-party legislature with a system dominant party is thus ((n-2)/n, 2/n·(n-1) … 2/n·(n-1)). Similarly, it is easy to see that the minimal integer representation in the same setting is (n/2, 1, 1 … 1) if n is even and ((n-1)/2 , 1, 1, … 1) if n is odd.

[18] $S_2 + S_3 \geq W$ implies $S_1 + S_3 \geq W$ and $S_1 + S_2 \geq W$

> **Proposition C1: Regardless of the number of parties in a top-three system, only the three largest parties can ever be pivotal.**[19]

> **Proposition C2: Any coalition excluding any two of the three largest parties in a top-three system is losing.**[20]

> **Proposition C3: The three largest parties in a top-three system are perfect substitutes for each other in the set of minimal winning coalitions**.[21]

By symmetry, therefore, the Shapley values and minimum integer weights (MIWs) of the top three parties must all be equal, and those of all other parties must be zero.

> **Proposition C4: $S_2 \geq W/2$ is a necessary condition for a top-three legislative party system.**[22] This is the third result focusing our attention on *W/2.*

Furthermore, if $S_1 + S_3 \geq W$ and $S_2 \geq W/2$, this implies $S_1 + S_2 + S_3 \geq 3W/2$. In other words the top three parties must between them control one and a half times the winning threshold in a top-three system, which can therefore *never arise when the winning quota is greater than two-thirds of total seats*.

The possibility of top-three party systems, which we show below are fairly common in postwar Europe, offers comfort to scholars working on formal models of legislative bargaining in "multi-party" systems. These models are often specified and solved for three-party systems, with more informal claims being made that results have relevance for the more general class of multi-party systems. Since almost no real legislature has precisely three parties, this might on the face of things seem disappointing. Without working through the interstices of any published formal proof, however, it seems at least possible that these may extend in an analytically tractable way to top-three party systems. This is because a top-three system is analogous, on some modeling assumptions, to a three-party system to which a set of "dummy" agents have been added who have no effect on play. This is another example of how our classification of legislative party systems might be theoretically helpful.

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[19] If P_2P_3 is winning then its complement, $(P_2P_3)'$, the coalition between P_1 and all parties outside the top three, is losing. Similarly, P_1P_3 winning implies $(P_1P_3)'$ losing, and P_1P_2 winning implies $(P_1P_2)'$ losing. No party outside the top three can render winning a coalition *excluding* two of the top three parties, since every such coalition must be losing. Yet, by definition of Type C, every coalition *including* two of the top three parties is winning regardless of the addition or subtraction of another party outside the top three.

[20] By definition S_1S_2, S_1S_3, and S_2S_3 are all winning, so their complements are all losing.

[21] This follows directly from the definition of a Type C legislature and Results C1 and C2.

[22] $S_2 + S_3 \geq W$, implies $S_2 \geq W/2$, since $S_2 \geq S_3$.

The empirical relevance of top-three systems arises, as we show below, because minimal winning coalitions (MWCs) are very much more likely to occur in Type C than in any other type of party system. Indeed, given the importance of the set of MWCs in many formal theoretical arguments, what stands out empirically is that *it is only in Type C systems that MWCs are the most likely type of government*.

Type D: "Top-two" party system

Top-two legislative party systems arise when the two largest parties can form a majority coalition ($S_1 + S_2 \geq W$) but P_1 and P_3 cannot ($S_1 + S_3 < W$). *The only two-party winning coalition is between the two largest parties*, since P_1P_3, the next-largest two-party coalition, is losing. Logically, this implies:

> **Proposition D1: One or other of the two largest parties in a top-two system is a member of every winning coalition.[23]**

However, unlike the situation in a top-three system, there are top-two systems that privilege the largest party. For example, it is possible for $S_1 + S_3 + S_4 \geq W$ while $S_2 + S_3 + S_4 < W$, giving P_1 more options that P_2.[24] Nonetheless P_1 and P_2 are at the "top" of a top-two party system in the sense that one or other of them must be a part of every legislative majority, while they and only they can form a winning coalition between themselves that excludes all other parties. Since $S_1 + S_3 < W$, we know $S_3 < W/2$ and since $S_1 + S_2 \geq W$, *we know* $S1 > W/2$[25]*;* indeed these are necessary conditions for a top-two party system. This is the fourth result focusing our attention on $W/2$.

Type E: "Open" systems

The defining inequality, $S_1 + S_2 < W$, of the residual class of "open" legislative party systems implies that there is *no winning two-party coalition* – since the two largest parties are not a winning coalition. It must also be true that $S_2 < W/2$; indeed this is a necessary condition for an open system. Logically, this implies:

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[23] Since P_1P_2 is winning its compliment is losing, Note therefore that Result D1 also applies to Type B and Type C systems.

[24] For example (51: 35, 20, 13, 12, 10, 10).

[25] Since $S_1 \geq S_2 \geq S_3$

Proposition E1: $S_1 < W/2$ is a sufficient condition for an open party system.[26] Every setting in which the largest party has fewer seats than half the winning threshold implies an open legislative party system.

If *W* is a simple majority, every election in which the largest party wins quarter or fewer of the seats gives rise to an open party system.

Proposition E2: An open party system and majority decision rule imply $N \geq 5$.[27]

In other words, it is necessary to model at least five-party systems to cover the full range of logical possibilities set up by the legislative arithmetic we outline in this paper. Crudely speaking, proof of a proposition about voting in legislatures that does not cover at least five-party systems may involve unexplored logical possibilities.

The theoretical significance of open systems arises because it is never possible for some party excluded from a winning coalition to tempt any one pivotal member of that winning coalition with an offer that can be implemented exclusively by those two parties, since any two-party coalition must be losing. This means that even the largest party must deal with *coalitions* of other parties – and in particular with potential collective action problems within such coalitions – in order to put together a winning coalition. To say more about such a setting we need a more explicit model of bargaining between parties and, in particular, of collective action within coalitions of parties. In all legislative party systems other than open systems, if the largest party does not win single-handed, it can win by forming a coalition with no more than one other party, at the very least the second-largest party. It can win without having to *coalesce with coalitions*.

The empirical significance of open legislative party systems arises, as we show below, because they are associated with significantly longer government formation negotiations, with significantly shorter cabinet durations, and with the formation of surplus majority or minority coalition cabinets.

Before turning in the next section from theoretical to empirical considerations, we pause to note that our partition of the universe of possible legislative party systems has a considerable bearing on how we might think about the legislative politics of particular *policy* decisions. Since it is not central to our argument, and since it requires us to be more precise about agent utility functions, we confine this discussion to an appendix.

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[26] $S_1 + S_2 < W$ implies $S_1 < W/2$ *since* $S_1 \geq S_2$

[27] A majority decision rule, N = 3 and $S_1 + S_2 < W$ imply $S_3 \geq W$. N = 4 and $S_1 + S_2 < W$ imply $S_3 + S_4 \geq$ W. Since $S_1 \geq S_2 \geq S_3 \geq S_4$, both implications are contradictions.

EMPIRICAL DISTRIBUTION OF PARTY SYSTEM TYPES

We now calculate the empirical distribution of types of legislative party system in 29 European parliamentary democracies during the period 1945-2010, using a dataset assembled by the European Representative Democracy (ERD) project (Andersson and Ersson 2012).[28] Winning coalitions in these *empirical* data are defined as those comprising a simple majority of legislators.[29] Specifying W to be a simple majority, we partitioned all 361 European post electoral party systems in the ERD data universe into our six (including B*) basic types. Figure 2 shows another way of representing, for minority legislatures, the exclusive and exhaustive partition of party systems specified in Figure 1.

The three left panels show regions defined by the seat shares of the three largest parties, which we used to classify types of party system. The boundaries of these regions, within what we can think of as a party system space, are specified by the inequalities set out in Figure 1.[30] For example, a lower region of the upper left hand plot is the exclusive preserve of "open" party systems, given the defining inequality $S_1 + S_2 < W$. A region of the lower left-hand plot is the exclusive preserve of "top-three" party systems given the defining inequality $S_2 + S_3 \geq W$ and our deduction that $S_2 + S_3 \leq 4W/3$ if W is decisive.

The right panels of Figure 2 map observed party systems in the ERD dataset into the theoretically possible regions identified in the corresponding left panels. Note in passing that empirical cases do not span the theoretically possible regions. Strikingly, second-largest parties we actually observe never win less than 10 percent of legislative seats, though this is perfectly possible theoretically.[31] Third-largest parties, again despite theoretical possibilities, also tend empirically to win more than 10 percent of the seats, except when second-largest and/or largest parties are close to majority status. The important empirical pattern we see in Figure 2 is that *regions close to the boundary conditions* between basic types of party system *are densely populated with empirical cases*. This implies that very small changes in the seat distributions of many empirically observed legislatures would flip them from one party system to another. We explore the considerable implications of this below.

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[28] For scrupulous documentation of coding protocols for this dataset, see http://www.erdda.se. Countries from the former Soviet bloc, as well as Spain, Portugal and Greece, were included after their first democratic election.

[29] Almost none of the theoretical conclusions elaborated above depended upon W being a simple majority. As is usual, however, winning coalitions in these *empirical* data are defined as those whose members comprise a simple majority of legislators. Any analyst with a better estimate of W in each of the countries concerned could of course re-run this analysis having re-specified the set of winning coalitions.

[30] Recall that, without loss of generality, $S_1 \geq S_2 \geq S_3$. The top left regions of each plot cannot be inhabited.

[31] There are many theoretically possible cases in which, for example, the largest party wins 45 percent of the seats and six or more other parties each win less than 10 percent. We do not observe these cases empirically.

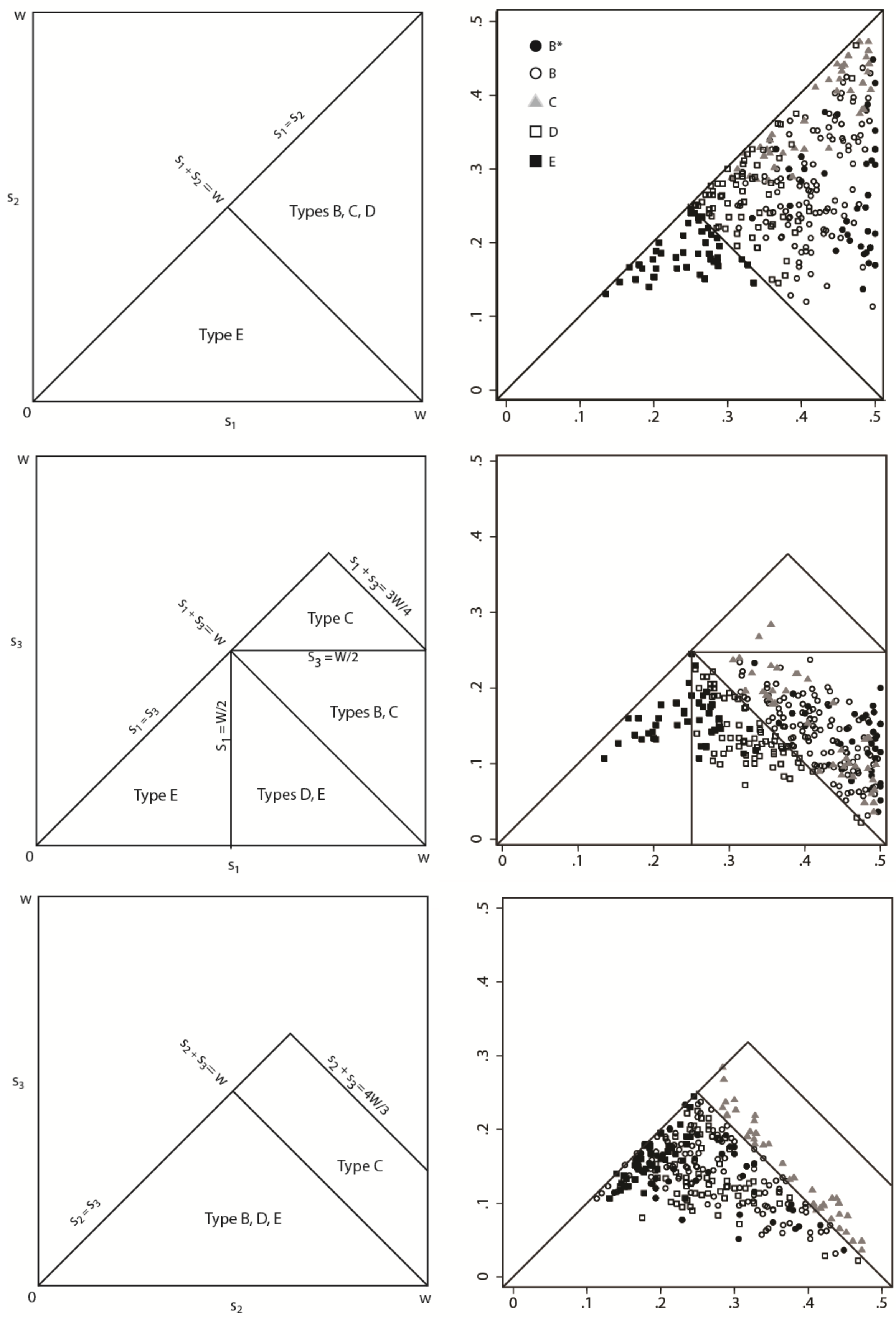

Figure 2. *Partition of party systems in theory (left panel) and as observed in postwar Europe (right panels).*

Table 2 reports the observed distribution of types, by the number of legislative parties, the average number of which is 6.6.[32] Most real systems (90%) with six legislative parties or fewer fall into the highly constrained types A to C. Most (57%) with seven parties or more fall into the relatively unconstrained types D and E, where the number of arithmetically possible majority coalitions on the table is very much greater and, in this sense, legislative politics is more complicated. Eleven percent of all party systems in post-war Europe fall into the top-three category; some had just three parties but the vast majority had more. There were top-two systems after 19 percent of post-war European elections, and unconstrained "open" systems after 12 percent of elections. The big news, however, is that there was a strongly- or system- dominant party in 41 percent of cases, while 11 percent had system-dominant parties. Strongly dominant parties are not just theoretical curiosities; they are a significant fact of real political life. Notwithstanding the PR electoral systems and resulting multi-party politics in most of postwar Europe, it is common to find legislative party systems dominated by one party able to play off the rest against each other.

	A	*B**	*B*	*C*	*D*	*E*	
Number of legislative parties	*Single party winning*	*System dominant party*	*Strongly dominant party*	*Top three*	*Top two*	*Open*	*Total*
2-6	47	37	64	35	18	1	202
	23%	18%	32%	17%	9%	0%	100%
7-16	19	2	43	4	50	41	159
	12%	1%	27%	3%	31%	26%	100%
All	66	39	107	39	68	42	361
	18%	11%	30%	11%	19%	12%	100%

Table 2. *Frequencies of legislative types in European legislative elections, 1945-2010.*

This empirical classification is important because, as we show below, different types of legislative party system are associated empirically with different political outcomes. Moving from the most constrained Type A systems to the least constrained Type E systems, it typically takes longer to form a government and the governments that do form tend to be more unstable. Furthermore, different types of legislative party system tend to be associated with different types of government.

[32] Standard deviation 2.63, median 6.

Figure 3 plots relative seat shares sizes of the three largest parties for empirically observed party systems. This shows that wide ranges of similar seat shares for each of the three largest parties are consistent with different legislative types. In other words, more than the seat shares *per se* it is precise relationships between seat shares of the top three parties, relative to our boundary conditions, that determine the basic type of legislative party system. Very similar seat shares across the top three parties can result in very different types of party system. This focuses our attention on the "fragility" of each realized party system – the probability that small random shocks to seats shares flip the system from one state to another.

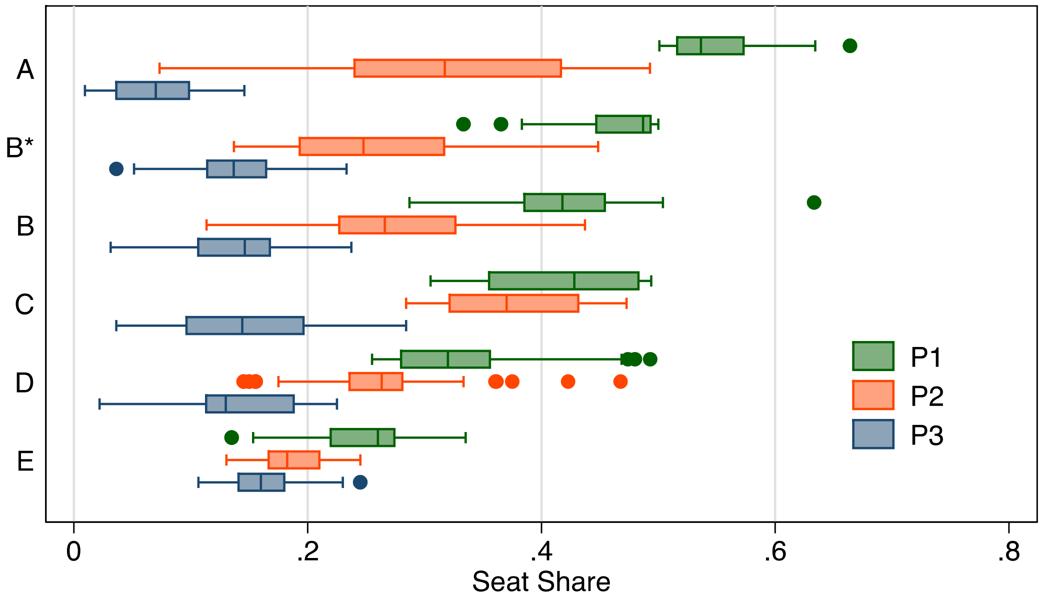

Figure 3. *Plots of S_1 - S_3 by legislative type: post-election party systems in the ERD Dataset.*

FRAGILITY OF LEGISLATIVE STATES

An important part of our argument is that, if the distribution of expected legislative seat shares following an election straddles one of our boundary conditions, the downstream legislative politics following an election can be something of a dice roll. Small random shocks, amplified in complex ways by electoral formulas and aggregations from constituencies that convert votes into seats, can have big effects. We simulate this using a simple and intuitive method to represent election results as random draws from an underlying distribution of expected results. We draw a new seat allocation for each party from a multinomial distribution where the proportions p_i are the actual seat share for party *i*, and *n* is the total number of seats.[33] This is very similar to the

[33] This means that parties who won no seats cannot win seats in any of the simulations, as p_i=0 for a party that won no seats. An alternative would be to use Laplace smoothing where we added one seat to each party, but we avoided

random disturbances added to model parameters by Laver and Shepsle (1998), to analyze the effects of critical events that might shock equilibrium conditions for observed cabinet portfolio allocations. By drawing new “shocked” seat allocations based on observed party seat shares, we generated a set of election results that might plausibly have been realized within a specified range of expected variance. Thus the Greek elections of May 2012 resulted in a Type D party system with 108, 52, 41, 33, 26, 21, and 19 seats held by seven parties. From our simulations given this distribution of legislative seat shares, this could have been realized as set of slightly different outcomes, resulting in different legislative types – for example, among many others:

D	104	56	43	43	20	13	21
B	112	52	51	29	29	13	14
B	109	48	48	32	25	21	17
B	113	50	39	33	37	16	12
E	102	44	46	36	34	20	18

The same election could plausibly have realized a Type D, B, or even a Type E party system – each with very different downstream political implications. In our simulations of the uncertainty around this particular observed outcome in Greece, a Type D party system was realized in only about 39% of simulated cases, with a Type B system being the most likely (52%) outcome. We estimated a very low probability that a Type B* or E system would have been realized. Our simulations of the “fragility” of the May 2012 realized outcome in Greece outcome are consistent with the argument that the June 2012 rerun of this election was in effect another random draw from the same stochastic process of seat distribution.

To simulate a range of “possible” distributions of legislative seats for every case in the ERD dataset – each consistent with the actual realized outcome – we drew 100 new elections for each observed seat allocation, and computed the legislative type associated with each possible outcome. The proportions of “shocked” legislative types associated with each observed legislative type are shown in Figure 4.

Most Type A party systems remained in Type A, though about 3-4 percent of these became each of Type B*, B, and C systems. The most common realization of a shock to a Type B* party system was to remain in Type B*, but about 25% became Type A systems with a single winning party, another 20% became Type B, and just under 10% became Type C. Type B party systems overwhelmingly stayed in Type B, although some became Type A or C party systems

this because it would change the number of parties in the system and potentially represent a different legislative dynamic.

(5% each) or Type B* or D systems (10%). Shocked Type C systems tended mostly (60%) to stay in Type C, though about 18% became Type B, 10% became Type B*, and 12% became Type A. About 60% of shocked Type D party systems stayed D, but about 25% became Type B, 10% became Type E, with a tiny number reaching Types A or B*. Shocked Type E party systems transited to Type D systems at a rate of about 20%, with about 4% becoming Type C.

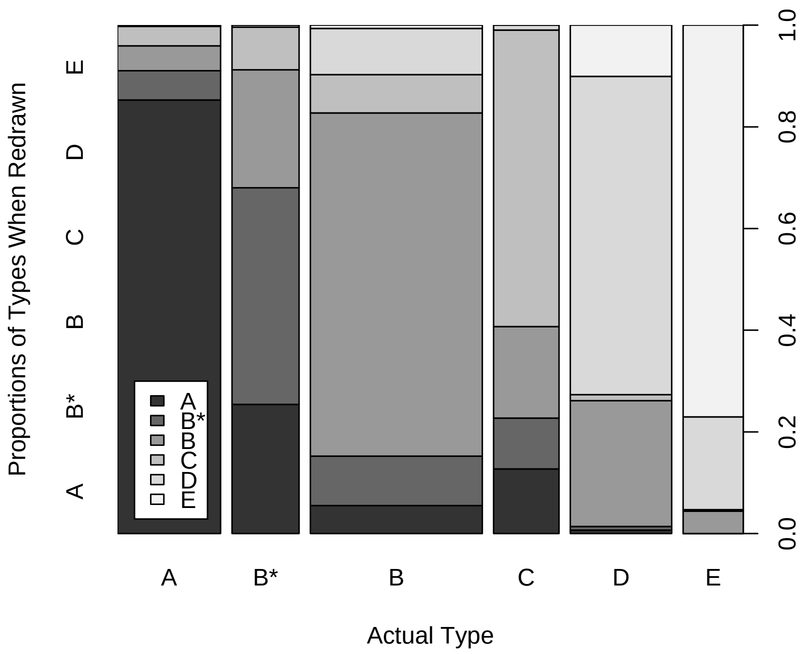

Figure 4. *Transitions from actual post-election governments to other legislative types, following simulated repeats of each election.* Each of 361 post-election governments was redrawn 100 using observed seat proportions from a multinomial draw, and the y-axis reflects the proportions by original type of each of the 36,100 simulated types.

Moving beyond the aggregate patterns reported in Figure 4, we now predict the particular legislative types resulting from small shocks to seat shares associated with each observed election result. To illustrate our core argument most clearly, Table 3 shows our predictions of changes in odds of flipping to each legislative type, given a change in the seat share of the *smallest* party – a party which is rarely the focus of attention in election polls or discussions of government formation. As control variables, we include differences between the seat shares of each of the top three parties and their closest competitor, to hold constant the main effects that determine legislative types.

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		Original Legislative Type				
		(1)	(2)	(3)	(4)	(5)
New Type	Variables	B*	B	C	D	E
A	P1 % Lead	**1.258**	**1.325**	**1.366**	**1.243**	
		[1.224 - 1.293]	[1.283 - 1.369]	[1.263 - 1.479]	[1.047 - 1.475]	
	P2 % Lead	**1.198**	**1.252**	**1.149**	**1.303**	
		[1.174 - 1.223]	[1.227 - 1.276]	[1.102 - 1.197]	[1.201 - 1.414]	
	P3 % Lead	**1.176**	**1.118**	0.932	1.321	
		[1.146 - 1.208]	[1.086 - 1.150]	[0.831 - 1.045]	[0.911 - 1.914]	
	P*n* %Δ	**0.782**	**0.749**	**0.856**	0.637	
		[0.741 - 0.825]	[0.680 - 0.826]	[0.743 - 0.986]	[0.350 - 1.159]	
B*	P1 % Lead		**1.022**	**1.252**	1.071	
			[1.007 - 1.037]	[1.193 - 1.314]	[0.976 - 1.175]	
	P2 % Lead		**1.036**	**0.925**	**1.151**	
			[1.027 - 1.046]	[0.906 - 0.943]	[1.102 - 1.202]	
	P3 % Lead		**0.929**	**0.666**	0.915	
			[0.910 - 0.949]	[0.631 - 0.704]	[0.709 - 1.180]	
	P*n* %Δ		1.025	1.308	**0.447**	
			[0.976 - 1.076]	[1.238 - 1.382]	[0.328 - 0.610]	
B	P1 % Lead	**1.097**		**0.849**	**1.069**	**1.149**
		[1.081 - 1.114]		[0.816 - 0.883]	[1.054 - 1.085]	[1.100 - 1.202]
	P2 % Lead	**1.031**		**0.913**	**1.058**	**1.13**
		[1.017 - 1.045]		[0.897 - 0.928]	[1.047 - 1.069]	[1.015 - 1.259]
	P3 % Lead	**0.916**		**0.835**	**1.18**	**1.484**
		[0.888 - 0.945]		[0.803 - 0.867]	[1.151 - 1.209]	[1.398 - 1.576]
	P*n* %Δ	**0.956**		**1.217**	**0.825**	**0.628**
		[0.918 - 0.995]		[1.164 - 1.273]	[0.783 - 0.869]	[0.525 - 0.751]
C	P1 % Lead	**0.783**	**0.82**		**0.827**	**0.427**
		[0.756 - 0.811]	[0.804 - 0.836]		[0.731 - 0.936]	[0.427 - 0.427]
	P2 % Lead	**0.978**	**1.034**		1.037	**0.022**
		[0.966 - 0.991]	[1.023 - 1.045]		[0.989 - 1.087]	[0.00270 - 0.179]
	P3 % Lead	**1.317**	**1.259**		**1.216**	**10.81**
		[1.259 - 1.378]	[1.230 - 1.289]		[1.100 - 1.345]	[9.383 - 12.46]
	P*n* %Δ	**0.954**	**0.783**		**0.474**	**0.279**
		[0.919 - 0.990]	[0.741 - 0.826]		[0.396 - 0.569]	[0.0309 - 2.519]
D	P1 % Lead	0.987	**0.924**	**0.586**		1.077
		[0.923 - 1.056]	[0.912 - 0.937]	[0.503 - 0.684]		[1.055 - 1.100]
	P2 % Lead	0.961	**0.986**	**0.808**		1.456
		[0.896 - 1.030]	[0.978 - 0.995]	[0.757 - 0.863]		[1.383 - 1.533]
	P3 % Lead	**0.623**	**0.901**	**0.695**		1.402
		[0.417 - 0.932]	[0.882 - 0.920]	[0.617 - 0.782]		[1.352 - 1.454]
	P*n* %Δ	0.996	**1.155**	**1.406**		**0.762**
		[0.841 - 1.180]	[1.107 - 1.206]	[1.278 - 1.546]		[0.697 - 0.832]
E	P1 % Lead		**0.936**		**0.897**	
			[0.887 - 0.988]		[0.879 - 0.917]	
	P2 % Lead		**0.603**		**0.745**	
			[0.518 - 0.703]		[0.723 - 0.767]	
	P3 % Lead		**0.83**		**0.785**	
			[0.756 - 0.911]		[0.755 - 0.816]	
	P*n* %Δ		1.173		**1.123**	
			[0.958 - 1.437]		[1.048 - 1.203]	
	Observations	3,900	10,000	2,700	5,900	3,500
	Log-likelihood	-4272.1183	-9665.0572	-2748.0535	-5111.8676	-1878.9035

Table 3. *Multinomial logistic regressions predicting simulated types from original legislative types.* All coefficients are exponentiated to represent risk ratios, relative to the original type as a baseline. 95% confidence intervals are in brackets, with bold coefficients statistically significant at the p<=.05 level. Data is the same as from Figure 4.

Our estimations in Table 3 report five multinomial logistic regressions, one for each empirically observed legislative type, except majority Type A party system.[34] To focus attention on key quantities of interest, we shade these in gray. To illustrate the interpretation of results from Table 3, consider the influence of a change in the seat share of the smallest party on the odds of becoming a Type D system – the system that Greece faced in May 2012. Look at the gray horizontal band of coefficients near the bottom of the table, associated with transitions to Type D party systems. This shows that a one percent change in the seat share of the *smallest* party increased the odds of a Type B party system becoming a Type D party system (thereby undermining the dominant position of the *largest* party) by 15.5%. The same shift in the smallest party seat share increased the probability a Type C party system transitions into Type D (thereby making parties outside the top three pivotal in majority coalitions) by 40.6%. While not every effect of changing the seat share of the smallest party had a statistically significant effect on the odds of changing the type of party system – thereby empowering or disempowering *other* parties in legislative bargaining – most such changes did. Small changes in the sizes of the smallest party can have big effects on legislative politics when no single party wins a majority.

Now consider the highlighted effect of changing the third largest party's lead over the fourth-largest party – again, not something that is a focus of attention for most election models or commentators. Table 3 shows that this also typically had both statistically and substantively significant effects on the probabilities of transition from one type of party system to another, even when relative positions of the largest three parties are held constant. Small shocks to the legislative party system, represented here by small seat share changes for small parties, substantially influence the legislative arithmetic and, as we now see, the types of downstream political outcome that might be expected.

TYPES OF LEGISLATIVE PARTY SYSTEM, TYPES OF POLITICAL OUTCOME

Types of legislative party system and the "difficulty" of forming a government

Rational politicians in a certain environment with complete information should negotiate equilibrium cabinets without delay: "… for the environments most interesting in policy-making applications, delay will almost never occur" (Banks and Duggan 2006).[35] It is well known, however, that some government formation negotiations drag out much longer than others. If the

[34] Each regression uses the original legislative type (before simulating a new seat allocation) as the base outcome, and reports exponentiated coefficients representing relative risk ratios, or the multiplicative change in odds of the stated outcome relative to the base category, for a percentage point change in seat share (or seat share difference).
[35] pp72-73

environment evolves stochastically, and/or if party leaders exploit private information (about personal preferences or which proposals their legislators will accept) bargaining delays may arise in equilibrium (Merlo 1997; Merlo and Wilson 1995). Diermeier and van Roozendaal apply this insight to government formation negotiations, and find a strong empirical relationship between their measures of uncertainty and durations of government formation negotiations (Diermeier and Van Roozendaal 1998). Martin and Vanberg, and more recently Golder, built on this work to confirm an empirical relationship between measures of uncertainty and government formation durations (Golder 2010; Martin and Vanberg 2003). Their strongest finding, reinforced using the ERD data in Table 4, is that formation negotiations immediately following an election tend to take much longer than those taking place between elections, following defeat or resignation of an incumbent.

Each of the studies we cite uses post-electoral government formation as an indicator of *uncertainty*. The rationale is that elections involve turnover of legislators, with less information about preferences of new legislators immediately after an election and more after the legislature has been is session. This seems plausible, but inter-electoral government formations are also distinctive for a very different reason with a direct bearing on bargaining delays. They are *endogenous* to legislative politics. Rational legislators terminate an incumbent government because they prefer some alternative. Inter-electoral cabinet formation negotiations may be shorter because there is an explicit candidate government at the outset, acceptable to the majority of legislators who terminated the former incumbent. This does not contradict the claim that inter-electoral government formations involve less uncertainty, but it does suggest a very different causal pathway for why such negotiations tend to be shorter.

Golder (2010) and others also associate longer government formation negotiations with more "complex" bargaining environments, measuring complexity in terms of the number and ideological polarization of parliamentary parties. This argument is also implicitly about uncertainty; more parties generate many more potential winning coalitions and thus many more possibilities to explore in an uncertain world. We argued above that different types of legislative party system are associated with different levels of complexity or "difficulty" in coalition formation. Moving from Type A to Type E systems, we move from the simplest setting, with a single majority party, through settings with a system dominant or strongly dominant party in the catbird seat, through "top-three" systems with only three pivotal parties no matter how many other parties there are, to the least constrained "top-two" and "open" systems with many pivotal parties and many possible majority coalitions. In an uncertain world, the latter cases imply more uncertainty, and our conjecture is that, as the complexity of coalition formation increases, so will the "difficulty" and hence duration of government formation. Table 4 shows mean durations of government formation negotiations, by type of system. The bottom row replicates previous findings that post-electoral negotiations last much longer (on average 39 days) than those between elections (13 days). The rightmost column supports our conjecture that mean durations of government formation negotiations should increase monotonically as the legislative arithmetic becomes less constrained.

Type of system	Post-election	Inter-election	All formations
A: Single majority party	20.3	8.1	15.7
	3.6	*2.7*	*2.5*
B*: System dominant party	24.9	2.9	17.2
	5.4	*0.9*	*3.8*
B': Strongly dominant party	32.6	16.1	25.0
	3.3	*2.1*	*2.1*
C: Top-three system	48.7	10.0	33.4
	7.7	*4.2*	*5.5*
D: Top-two system	46.5	18.5	34.0
	4.9	*5.6*	*3.9*
E: Open system	72.3	12.7	36.3
	7.0	*2.0*	*4.2*
All formations	38.6	13.3	27.1
	2.2	*1.4*	*1.4*

Table 4. *Mean durations of government formation negotiations in postwar Europe, by type of legislative party system.* Standard errors in italics. Formation durations data, taken from the ERD dataset, count days between election/government resignation and investiture of new government.

Previous authors used Cox proportional hazards survival models to analyze government formation delays. Creating binary variables for legislative types, we use the proportional hazards model specified by Golder (2010) to investigate whether these types do indeed distinguish between systems in terms of bargaining delays during government formation. We follow Golder in using the number of legislative parties as an indicator of uncertainty, in controlling for the existence of a single majority party, and in distinguishing between post-electoral and inter-electoral formations. Rather than following earlier scholars and using the highly subjective and potentially endogenous notion of "positive parliamentarianism" as a factor contributing to the difficulty of government formation, we use the related but objective and binding constitutional constraint of a constructive vote of no confidence. *Inter*-electoral government formations should be much quicker with a constructive vote of no confidence, since the next government must be explicitly identified in the no confidence motion that ends the previous administration. The constructive vote of no confidence should however have no effect on *post*-electoral formations.[36] Unlike the dataset used by Golder, which is confined to Western Europe and ends in 1998, the ERD dataset ends in 2010 and includes 10 former communist countries in Central and Eastern Europe. We therefore include a CEE dummy. Especially at the beginning of their experience as democracies, party systems in CEE post-communist states were very new, leading us to expect greater uncertainty, hence longer bargaining delays, in CEE countries.[37]

Table 5 shows Cox proportional hazards estimates of the effects of independent variables on durations of government formation negotiations in postwar Europe. Rather than following Golder and using interaction terms to capture effects of key independent variables, conditional on whether negotiations follow an election, we estimate different models for post-electoral and inter-electoral settings, since these differ in many ways relevant to government formation.

Model 1 is a stripped-down benchmark. It replicates findings from previous work that increasing the number of parties, which has an exponential effect on the number of winning coalitions and hence on the amount of information needed to take every possibility into account, reduces the hazard rate and thereby increases typical durations of government formation negotiations.[38] This effect is essentially the same in post- and inter-electoral negotiations. As expected, a constructive vote of no confidence significantly shortens *inter*-electoral formation negotiations[39], but has no significant effect on *post*-electoral negotiations. Former Communist states do have longer negotiations in inter-electoral settings, but not immediately after elections. Model 2 replaces the simple distinction between systems with or without a majority party with the different types of legislative party system specified in Figure 1, using single party majority systems as the baseline. Coefficients for other independent variables are essentially unchanged.

[36] If we include the ERD variable for positive parliamentarianism in models that also include the constructive vote of no confidence, it has no significant effect on bargaining delays. It has the effects observed by Golder if the no-confidence variable is dropped.

[37] Golder included a measure of ideological polarization as another indicator of bargaining difficulty. When we included the ERD measure of ideological polarization, however, we found no significant effect, and therefore excluded it from the analysis we report here.

[38] Diermeier and van Roozendal (1998) used the *effective* number of legislative parties in this context, but Golder uses the absolute number of legislative parties. It is this latter number that has a direct effect on the number of winning coalitions. We also agree with Golder that it is not a good idea to use the number of parties *in government*, as do Martin and Vanberg (2003); this is clearly endogenous to government formation negotiations.

[39] The hazard rate on negotiation duration is positive and significant.

	Model 1		*Model 2*		*Model 3 (country fixed effects)*	
	Post-election	*Inter-election*	*Post-election*	*Inter-election*	*Post-election*	*Inter-election*
Number of parties	-0.10** (0.02)	-0.14** (0.02)	-0.08** (0.03)	-0.13** (0.03)	-0.01 (0.04)	-0.03 (0.04)
Constructive vote of no-confidence	-0.14 (0.12)	0.85** (0.22)	-0.11 (0.11)	0.94** (0.23)	0.79 (0.63)	1.84** (0.44)
CEE country	-0.10 (0.11)	-0.59** (0.16)	-0.11 (0.14)	-0.60** (0.15)	-1.19 (0.74)	-3.62** (0.79)
Minority parliament	-0.51** (0.21)	-0.55** (0.17)				
B*: System-dominant party			-0.23 (0.32)	0.45 (0.28)	-0.49 (0.30)	0.10 (0.26)
B': Strongly-dominant party[40]			-0.31 (0.21)	-0.28 (0.26)	-0.64** (0.25)	-0.26 (0.22)
C: Top-three system			-0.94** (0.27)	-0.24 (0.33)	-0.42 (0.31)	-0.68** (0.25)
D: Top-two system			-0.65** (0.22)	-0.14 (0.27)	-0.70** (0.27)	-0.06 (0.33)
E: Open system			-0.90** (0.23)	0.09 (0.29)	-1.20** (0.32)	0.03 (0.32)
Log likelihood	-1572	-1193	-1562	-1228	-1446	-1172
Observations	331	266	331	272	331	272

Table 5. *Cox proportional hazards models of durations of government formation negotiations in Europe, 1945-2010.* Classifications of party systems by the authors; all other data from the ERD dataset.

[40] Systems labeled B' in have a strongly dominant party that is not system dominant.

Types of legislative party system have the predicted effects on durations of *post-electoral* formation negotiations. These do not take significantly longer in systems with system-dominant and strongly dominant parties than in those with single majority parties.[41] In contrast, there are significantly longer formation delays in Type C, D and E systems. Note in particular that, while our classification of party systems is affected strongly by the number of legislative parties, effects of party system types on bargaining delays are measured holding the number of parties constant. In contrast, differences between types of legislative party system have no systematic effect on the duration of *inter-electoral* government formation negotiations. This is consistent with Golder's argument that inter-electoral formations are high-information settings, so that the different information requirements posed by different types of party system do not bite. It is also consistent with the view that a putative alternative government is typically on the table in inter-electoral formation settings, so that all coalition possibilities are less likely to be explored. Either way, the Model 2 estimates show post- and inter-electoral government formations to be completely different. Conventional arguments about government formation seem to apply to post-electoral negotiations, but not necessarily to those taking place between elections.

Model 3 replicates Model 2, but adds a full set of country fixed effects, to eliminate the possibility that different countries tend to have different types of party system, with government formation negotiations tending to last longer in some countries as result of unmodeled differences between countries.[42] Our classification of legislative party systems, if it adds value, should pick up significant variation between different types of system within the same country. Country fixed effects soak up the impact of the number of legislative parties[43] but not the impact, in inter-electoral formations, of a constructive vote of no confidence or former-communist status. Most of the impact of party system types on *post-electoral* negotiations is robust to the addition of country fixed effects. Systems with system dominant parties still do not have significantly longer formation negotiations than those with majority parties; Type D and Type E systems still do have significantly longer formations. The differences are that Type B systems, with strongly dominant parties, do have longer bargaining delays when country fixed effects are added, and top-three systems do not. All coefficients are in the predicted direction. The non-effect of our party system types on *inter-electoral* formation durations is also essentially robust to adding

[41] Though the non-significant effects are in the "right" direction, with formation negotiations tending to be longer than in Type A systems.

[42] Luxembourg, close to the overall mean for government formation negotiations, is the excluded category. We do not report substantive effects, many of which are significant since these simply reflect deviations of individual countries from the base category.

[43] The previous estimate of this effect could thus be a result of the fact that different countries tend to have different numbers of parties and also, for unmodeled reasons, to have different bargaining delays.

country fixed effects. Table 4 underwrites the pattern summarized in Table 4. Our legislative types do classify post-war European party systems according to the "difficulty", measured as the duration of negotiations, of forming governments in minority parliaments.

Types of legislative party system and types of government

Different types of legislative party system are also associated with different types of coalition cabinet in minority situations. Most theoretical and empirical accounts of government formation in parliamentary democracies imply that, if one party wins a parliamentary majority, it goes on to form the government.[44] For situations in which no one party controls a majority, the overwhelming norm in postwar Europe, government survival depends upon coalitions of legislative parties. This gives rise to different types of executive, depending on the strategic setting. The most basic theoretical and empirical distinction is between:

- minimal winning coalitions (MWCs), winning coalitions made losing by defection of *any* member;
- surplus coalitions, which include at least one member whose defection leaves the coalition winning;
- minority cabinets, which comprise one or more parties that do not between them control a majority.

Models assuming politicians to be motivated only by private benefits of office tend to imply MWCs. Models that assume politicians are motivated by preferences over public policy outcomes may also imply minority or surplus majority cabinets (Laver 1998). There is also an informal folk-wisdom that surplus cabinets provide insurance against defections in times of high uncertainty or low party discipline (Laver and Schofield 1998). Table 6 classifies the postwar European governments in the ERD dataset that were formed in minority situations into MWCs, minority and surplus majority cabinets,[45] further classifying minority governments into coalition and single party cabinets. It shows a striking relationship between type of legislative party system and type of government.[46] Recall that top-three systems are the closest real-world analogue to the "three-party" systems of many formal models which, if they assume office-seeking politicians, tend to predict MWCs. Table 6 shows that, within the class of real top-three

[44] This assumes, as we do here, that high levels of party discipline preclude the possibility of the majority party splitting during the government formation process.
[45] This includes all governments, not just those forming immediately after an election.
[46] We have specified type B systems as supersets of type B* systems. In this table and all that follow, however, we create and exclusive and exhaustive partition of systems by dividing type B into types B* and B'. Type B' is a type B legislature that is not B*.

legislative party systems, MWCs are indeed the norm. Conditional on observing a top-three system, theoretical predictions of MWCs are typically vindicated. We saw from Table 2, however, that top-three party systems only arise after 11 percent of postwar European elections. Table 6 restates the well-known empirical pattern that only about one-third of all governments arising from post-war European minority systems are MWCs, while about two-thirds are either minority cabinets or surplus majority coalitions (Gallagher et al. 2012). Notwithstanding many theoretical models, MWCs are *not* the norm in real parliamentary settings and our classification of legislative party systems throws light on why this might be the case.

Cabinet type	*B* System dominant party*	*B' Strongly dominant party*	*C Top three*	*D Top two*	*E Open*	*Total*
MWC	24	68	48	26	28	194
Single party minority	29	62	7	16	5	119
Minority coalition	3	29	3	33	21	89
Surplus	4	32	1	42	38	117
Total	60	191	59	117	92	519

Table 6. *Types of government forming from minority settings in Europe, 1945-2010.*

First, note from Table 6 that minority administrations are the most common type of government in Type B* and Type B party systems. Over half of real parliaments with a system-dominant party, and nearly half of those with a strongly-dominant party, generate minority governments, typically comprising the single largest party. Without getting into fine print of any particular model of government formation, this reflects the plain fact that system-dominant parties in particular, and strongly-dominant parties more generally, participate in most winning coalitions, while few winning coalitions exclude them. This has the implication that, as other constraints are brought to bear upon government formation negotiations, whether these be squalid personal animosities, lofty policy disagreements, or anything in between, it can quickly happen that all winning coalitions excluding the dominant party become infeasible for one reason or another. This leaves the dominant party able to form a minority government because no feasible winning coalition agrees on an alternative. Considering potential policy disagreements, furthermore, recall the high probability that system- or strongly-dominant parties are pivotal on an arbitrary policy dimension, dividing policy-motivated opponents and facilitating the formation of a minority administration.

Turning to surplus majority cabinets, Table 6 shows these to be most common in the type D and type E party systems which, as we have seen, tend to sustain many more possible winning coalitions. If we assume that uncertainty about which coalition deals might or might not work increases with the number of different winning coalitions, such uncertainty increases in the relatively unconstrained Type D and E party systems. The prevalence of surplus majority coalitions in these thus comports with the folk-wisdom that surplus majority governments are responses to high levels of uncertainty whereby politicians insure against future intra-coalition disagreements by taking on surplus members, so that the government cannot be brought down by individual defections, or be held ransom by the threat of these.

Overall, the striking patterns in Table 6 are that: Type B and B* systems dominated by the largest party tend to generate minority cabinets; “three-pivotal-party” negotiations in Type C systems tend strongly to generate minimal winning coalitions; and the less constrained and arguably more uncertain negotiations found in Type D and E systems are associated with surplus majority cabinets.

Types of party system and typical government durations

Once a cabinet has taken office in a parliamentary democracy, a key question concerns how long it will last, in a setting where any government can at any time either resign or be dismissed by a majority vote of no confidence. There is a substantial political science literature on government stability and it is not feasible to review or extend this here (Diermeier and Stevenson 1999, 2000; King et al. 1990; Laver and Shepsle 1998; Lupia and Strom 1995; Warwick 1994; Browne et al. 1986). Out argument here, in the context of this literature, is that typical cabinet durations differ significantly between different types of legislative party system. Table 7 shows the bottom line: governments do tend to last longer in the most constrained Type A and Type B* systems, and less long in Type E systems where the number of winning alternatives to the incumbent is highest.

Moving beyond a simple table, we can deploy the type of Cox proportional hazards approach used above to model bargaining delays, taking account of key findings in the government termination literature. First, given the convention of regarding governments as terminating whenever there is a general election, government durations are treated as “censored” if they are brought to an “artificial” end by a scheduled election, and might otherwise have lasted longer. The data show a big spike in durations at about 1400 days (about 46 months), given the typical constitutional inter-election period in such countries of four years. Accordingly, government durations over 1350 days are treated as censored.

Type of legislative party system	*Post-election*	*Inter-election*	*All cabinets*
A: Single majority party	1082 *59*	552 *61*	891 *51*
B*: System dominant party	942 *71*	509 *74*	786 *59*
B’: Strongly dominant party	831 *52*	451 *36*	652 *35*
C: Top-three system	987 *91*	425 *85*	775 *74*
D: Top-two system	929 *55*	346 *41*	676 *45*
E: Open system	695 *77*	289 *31*	455 *41*
All formations	**909** *27*	**414** *20*	**688** *20*
Minimal winning cabinets	1034 *43*	528 *48*	875 *37*
Single-party minority cabinets	735 *57*	373 *42*	568 *40*
Minority coalition cabinets	659 *78*	315 *41*	451 *43*
Surplus majority cabinets	774 *58*	414 *37*	587 *36*
Non-CEU	936 29	431 24	726 23
CEU	761 63	362 31	534 40

Table 7. *Mean government durations, in days, by type of party system and cabinet.*
Standard errors in italics.

This bears upon a second issue, which is the distinction between post- and inter-electoral government formations. Governments forming between elections have lower potential durations than governments forming immediately after elections. In addition, as noted above when discussing bargaining delays, governments formed between elections are negotiated in settings where a previous equilibrium cabinet has been destabilized, and where rational politicians presumably had an alternative in mind when bringing down the incumbent. For this reason, in addition to treating durations over 1350 days as censored, we consider only the durations of governments forming immediately after an election.[47] The empirical work cited above shows that the type of coalition cabinet in a minority setting has a significant bearing on its expected duration, as does the "complexity" of the bargaining environment in which it is set. Our types of legislative party system capture the complexity of the bargaining environment, but the stripped down benchmark model uses the number of legislative parties to measure this.[48] In relation to the relationship between cabinet types and government durations in minority settings, Table 7 clearly shows that the key distinction is between minimal winning cabinets and others, be they minority or surplus majority administrations. Accordingly, we control for cabinet type using a binary variable for whether or not the cabinet is minimal winning. Finally, the ERD dataset we use here includes governments in post-communist CEU democracies, whereas previous empirical work focused exclusively on Western Europe. We already assumed more uncertainty in the relatively new party systems of the post-communist CEU, and Table 7 confirms that governments tend systematically to last less long in the CEU. Accordingly, we include a binary control for whether the cabinet was in a CEU country.

Table 8 reports Cox proportional hazard estimates for three models of durations of governments formed after elections in postwar Europe. Model 1 is a stripped-down benchmark, using the absolute number of legislative parties to measure the complexity of the bargaining environment, an MWC dummy to control for cabinet type, and a CEU dummy to identify the less-established post-communist party systems. Increasing the number of legislative parties, and hence the number of possible legislative coalitions, does significantly increase the hazard of a government termination, as does the fact that the cabinet is in a CEU country. Minimal winning coalitions are estimated to have lower probabilities of termination, holding other factors constant, though this coefficient is not statistically significant.

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[47] Diermeier and Stevenson (1999, 2000) take a different approach to the same, measuring the competing risks of scheduled and unscheduled terminations. Both approaches share the view that it is the unscheduled terminations that convey more information.

[48] Previous scholars typically use the effective number of parties in this context but, for reasons noted above, we feel the absolute number of parties, which has a direct and exponential effect on the number of possible coalitions, is a better measure of complexity.

	Model 1	Model 2	Model 3 (country fixed effects)
Number of parties	0.17** (0.06)	0.02 (0.07)	-0.32* (0.16)
CEU country	1.21** (0.29)	0.93** (0.36)	1.55* (0.76)
Minimal winning coalition	-0.44 (0.24)	-0.40 (0.24)	-0.35 (0.39)
B*: System-dominant party		-1.32* (0.67)	-4.03* (1.62)
B: Strongly-dominant party		-1.33** (0.44)	-3.57** (1.32)
C: Top-three system		-2.03** (0.66)	-4.71** (1.50)
D: Top-two system		-0.82* (0.38)	-3.08* (1.28)
Log likelihood	-213	-209	-173
Observations	279	279	279

Table 8. Cox proportional hazards models of post-electoral cabinet durations in European minority settings, 1945-2010.* Considered censored at 1350 days.

Model 2 adds binary variables for our types of legislative party system, treating the least stable Type E system as the excluded type in minority settings. Proportional hazards estimates for these are all significant and negative, showing that each party system type is associated with a lower hazard rate (cabinets of longer duration) than those in Type E. As Table 7 suggests, the big difference in cabinet durations is between cabinets forming in Type E, open, systems and the rest. Model 3 adds a full set of country fixed effects, and shows that the lower hazard rates of cabinets in non-type E systems are robust to this.[49]

[49] We treat Finland, of the 29 countries the one with mean durations closest to the overall mean, as the excluded category.

CONCLUSIONS

Despite the profusion of theoretically possible seat distributions that could arise after any legislative election in a multiparty system, legislative party systems fall into a much smaller number of theoretically relevant equivalence classes. One set of these generates a mutually exclusive and exhaustive partition of the universe if possible seat distributions into five "types" of legislative party system (Figure 1). We show that these types of party system differ from each other in theoretically significant ways. For example, in a Type B system with a dominant party, the largest party, and only the largest party, is a member of every two-party winning coalition. In a Type C system, no party outside the largest three is pivotal in any winning coalition. There is no two-party winning coalition in a Type E system, the only type of party system not subject to the arithmetic constraints we identify, and which must comprise at least five parties.

We classify postwar European party systems according to our exclusive and exhaustive partition, and show that regions of the "party system space" close to critical boundary conditions between types are densely populated (Figure 2). Any legislative election is subject to stochastic processes, so that the result is in effect a random draw from a distribution of expected seat distributions. If this distribution straddles a key boundary condition, as Figure 2 implies it often does, different random draws from the same underlying distribution may well flip the resulting real party system from one state to another with theoretically critical effects. For example, as party systems flip stochastically into and out of Type C, a set of parties outside the top three flip into and out of a situation in which they are pivotal in winning coalitions, with substantial consequences for legislative bargaining.

We also show that our exclusive and exhaustive partition of legislative party systems is of more than hypothetical interest. Differences between types of party system have substantial effects on: how long it takes to form a government (Tables 4 and 5); the type of government that eventually forms (Table 6); and the typical duration of the government that does form (Tables 7 and 8).

Insights that might be derived from our partitioning of legislative party systems are "model free", logical implications of the basic arithmetic of legislative voting. They do not depend on utility functions of key agents. They apply whether legislators are motivated by perks of office, by public policy preferences, by spite envy and greed, or by anything else – provided they seek to realize these objectives by forming winning coalitions in the legislature. They apply notwithstanding detailed institutional structures that circumscribe legislation or government formation. Such institutions may make a huge difference, but the basic legislative arithmetic imposes its own severe constraints on what can happen. The constitution may specify that the President nominates the Prime Minister, as in France. It may, as in Greece, stipulate that party leaders lead government formation negotiations in strict order of party size. Notwithstanding such important institutional factors, the basic legislative arithmetic still applies. Proposals must still win legislative votes, and the constraints imposed by our boundary conditions still bite. While particular well-specified models of legislative bargaining and/or government formation may well *further* constrain the set of outcomes implied by the basic legislative arithmetic, they cannot *transcend* this.

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APPENDIX: PARTY SYSTEMS AND POLICY DECISIONS

Assume legislators vote on particular issues, that possible positions on each issue can be placed on a single latent policy dimension. Assume that, for any issue under consideration, legislators have an ideal point on the latent dimension concerned, and a component of their utility function that declines monotonically as the policy agreed by the legislature moves further way from this. Differences between the types of party system set out above bear in striking ways upon the policy outcomes that might emerge in such a setting, because our boundary conditions impose different binding constraints on the identity of the party occupying the pivotal position on an arbitrary issue dimension – a dimension for which we are ignorant *a priori* of the ordering of party positions. First note that, if a party is pivotal to no legislative majority, it can never be in the pivotal position on any particular issue dimension.[50] This is why our classification of legislative party systems bears directly on legislative voting on policy issues.

In Type B* systems the *system*-dominant party, while not winning on its own, can form a winning coalition with any other party. It must therefore occupy the pivotal position on any issue dimension for which there is a party on either side of it. Logically, this implies:

> **Proposition B5*: A system-dominant party must be at the pivotal position on any issue dimension for which it is not at one of the most extreme party positions. If *P*** is at the extreme of some issue dimension, then the pivotal party must be adjacent to *P***.**[51] The *a priori* probability that a system dominant party in an *n*-party system is pivotal on some arbitrary issue dimension under consideration by the legislature is therefore *(n-2)/n*.

Even when there is an indefinite number of unknown issue dimensions that might form the basis of legislative decisions, therefore, *the pivotal party on any issue is either the system-dominant party or the party adjacent to it, regardless of the positions of all other parties*. A system dominant party therefore has substantial control over legislative policy outputs.

In Type B systems a strongly-dominant party, *P**, can form majority coalitions with both P_2 and P_3, which implies:

> **Proposition B5: If P_2 and P_3 are on opposite sides of *P** on some issue dimension, then *P** is at the pivotal position, regardless of the positions of all other parties.**

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[50] Note also that, taking at set of issue dimensions together and treating these as a multidimensional issue space, parties may occupy strategically important locations by virtue solely of their issue positions. However, leaving aside the possibility of log-rolling, when legislatures dispose of one issue at a time it remains true that a party pivotal to no legislative majority can never be pivotal on any issue dimension under consideration.

[51] Since P_1** can form a winning coalition with any other party

This gives a P^* a somewhat privileged position in affecting legislative policy outputs, though clearly less than that enjoyed by a P^{**}. In Type C, top-three systems, no party outside the top three can be pivotal, so the pivotal party on any conceivable policy dimension must be to one of the three largest parties. Logically, this implies:

> **Proposition C5: The pivotal party on any issue dimension must be the most central of the top three, regardless of the issue positions of the smaller parties.**[52]

In Type D, "top-two", party systems, it follows logically that:

> **Proposition D2: The pivotal party on any issue dimension must be located on the interval between P_1 and P_2, regardless of the positions of smaller parties.**[53]

This is much less a constraint on the location of the pivotal party on an arbitrary issue dimension than in the previous three settings. Indeed if P_1 and P_2 are at opposite ends of some issue dimension, it is no constraint at all. In Type E "open" systems, the defining inequality, $S_1 + S_2 < W$, implies that that all two-party coalitions are losing. This imposes no constraint of substance on the location of the pivotal party on an arbitrary issue dimension.

The results set out above highlight a stronger relationship than might hitherto have been suspected between constant sum bargaining in legislatures over a fixed set of perquisites and variable sum bargaining over policy. The reason for this is that *the identity of the pivotal party on an arbitrary policy dimension in a weighted voting game is determined as much if not more by party sizes as by party policy positions***.** One consequence of this is that the normalized Shapley value, typically seen as applying to constant sum bargaining over a fixed cake, has a precise interpretation in terms of variable sum legislative bargaining over policy. The normalized Shapley value of party P is the proportion of all orderings of coalition formation in which P is pivotal. This means that it is also the proportion of all orderings on an arbitrary policy dimension in which P is pivotal. The Shapley value of party P, therefore, is precisely the probability that P is pivotal on an arbitrary policy dimension. In this sense, the Shapley value has an intuitively meaningful interpretation in terms of legislative bargaining over public policy.

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[52] For any top-three party that is not the most central on some issue dimension, there must be a winning coalition of the two other top-three parties, either to the right or the left of it. Therefore the non-central top-three party cannot be pivotal on this dimension.

[53] Since P_1P_2 is a winning coalition, the pivotal party on any issue dimension cannot be either to the left or to the right of both P_1 and P_2.